

# On the entropy solution to an elliptic problem in anisotropic Sobolev-Orlicz spaces

Kozhevnikova L.

Kazan Federal University, 420008, Kremlevskaya 18, Kazan, Russia

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## Abstract

© 2017, Pleiades Publishing, Ltd. For a certain class of anisotropic elliptic equations with the right-hand side from  $L_1$  in an arbitrary unbounded domains, the Dirichlet problem with an inhomogeneous boundary condition is considered. The existence and uniqueness of the entropy solution in anisotropic Sobolev-Orlicz spaces are proven.

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## Keywords

anisotropic elliptic equation, entropy solution, existence of solution,  $N$ -functions, Sobolev-Orlicz space, uniqueness of solution

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